ALGEBRA AND TOPOLOGY
HOMEWORK ONE
DUE: 7/25

There are two different types of labels: alphabets and numbers. You only need to write up your solutions to those exercises labelled by alphabets. The rest is for fun.

Exercise A. The quantity $F - E + V$ introduced in class (for a sphere) is called the *Euler characteristic* (for a sphere). Find the Euler characteristic of an annulus and a doughnut. First, you have to check that $F - E + V$ is well-defined, that is, it doesn't depend on how you construct them.

Exercise 1. Find all integral solutions to the Diophantine equation

$$\frac{1}{m} + \frac{1}{n} - \frac{1}{2} = \frac{1}{E}$$

with $m, n \geq 3$ and $E > 0$.

Exercise 2. Consider a polyhedron P. Denote by F_n the number of n-gon faces in P and V_n the number of vertices in P where exactly n edges meet. Compute

$$(F_3 - F_5 - 2F_6 - 3F_7 - \cdots) + (V_3 - V_5 - 2V_6 - 3V_7 - \cdots).$$

Exercise B. Show that the composition of two continuous maps is continuous, using the definition in class.